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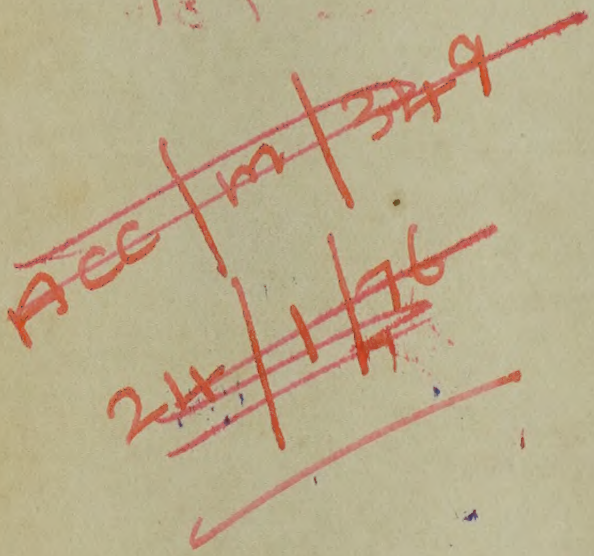

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WITH HINTS ON THE METHOD OF SOLVING RIDERS

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BY

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PREFACE.

NEARLY twenty years ago I wrote and subsequently published a little book of Rider Papers on Euclid for use in my own classes. The book was widely used, but the recent changes made in teaching Geometry have rendered the book useless in most schools. I have, therefore, entirely re-written and extended the papers, retaining only so much of the old book as experience had shown me to be of value.

The new papers have been carefully arranged and graduated in difficulty. They are divided into fourteen Sections. Sections I. to VII., containing Papers 1 to 42, deal with angles, triangles and parallelograms; Section VIII. deals with rectangles and squares; Sections IX. to XIII., containing Papers 49 to 78, deal with the properties of the Circle; and Section XIV. contains six papers dealing chiefly with similar figures. These Sections are so constructed that they can be taken in different orders, and will be found to suit almost any text-book of Geometry.

Each Paper contains six questions, arranged in two sets of three. The first question in each three is a problem in Geometrical Drawing; the next two questions are usually riders. My method of using these papers was to set a paper for home work once a week. I looked over the pupils' written answers, and then worked on the black-board any questions which had not been satisfactorily answered by the majority of the pupils. This occupied about 15 minutes. If the whole of a paper is considered

too much for one lesson, half a paper can be set, or half the class can take Questions 1, 2, 3, and the other half of the class can attempt Questions 4, 5, 6. It will be found that the first half of each paper is usually easier than the second half of the same paper, and the latter should, therefore, be assigned to the cleverer pupils. A similar plan can be adopted, if thought desirable, with whole papers, for they have often been drawn up in pairs, questions in one paper corresponding or leading to questions in the next paper. Or again, Sections III., V., VII., X. and XII. can be treated as alternative Sections, being based on the same Problems and Theorems as the Sections that immediately precede them. Here, too, the later questions are the more difficult.

At the beginning of each Section I have printed the Enunciations of all the Problems and Theorems assumed and used in the Section, and I have assigned numbers to these Enunciations. By this means the book can be used for the purposes of school examinations. If the numbers of certain Enunciations and certain questions are dictated to the pupils for them to answer, there will be no need to provide them with printed examination papers.

These devices have been suggested to me during my many years' experience as a teacher of Geometry. The riders I have selected are generally important propositions and such as are needed in the future study of Mathematics. I hope, therefore, that the use of this little book will serve to lighten the labours both of teachers and of pupils.

RUPERT DEAKIN.

King's Norton, 1907.

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HINTS ON THE METHOD OF SOLVING RIDERS.

1. Read the rider through carefully and thoughtfully, so that as much as possible of the question may remain fixed in your memory.

2. Notice all the geometrical terms used in the rider and think over the exact meaning of each of these terms.

3. Draw a rough figure to illustrate the rider.

4. Write down all that you are allowed to take for granted, and what you are required to prove. In doing this be careful not to assume more than you are allowed, and do not omit any fact that is given in the rider.

5. Think of any proposition or rider that at all resembles the one you are attempting to solve, and try to solve your rider in the same way.

6. Try to substitute for any given term or part of the rider any fact or facts that can be deduced from the given term or part of the rider.

7. Assume that the rider is true and try to work back from the conclusion to some known problem or theorem. Reversing this work will probably give you the required proof of the rider.

8. Having tried these methods, draw a neat and accurate figure, moderately large in size; write out again what can be taken for granted and what has to be proved; and add as much of any part of the solution as you can.

9. Do not spend too much time, and, as a rule, not more than ten minutes, over any one rider.

SECTION I.

The Exercises and Riders in Section I., Papers 1 to 6, are based on the following Problems and Theorems :

PROBLEMS.

1. To bisect a given rectilineal angle.

2. To bisect a given finite straight line.

3. To draw a straight line perpendicular to a given straight line from a given point in the given line.

4. To draw a straight line perpendicular to a given straight line from a given point without the given line.

5. To construct an angle equal to a given rectilineal angle.

6. To construct a triangle from simple data.

THEOREMS.

1. If a straight line stands on another straight line, the sum of the two angles so formed is equal to two right angles.

2. If at a point in a given straight line, two other straight lines on opposite sides of it make the adjacent angles together equal to two right angles, then these two straight lines are in one and the same straight line.

3. If two straight lines intersect, the vertically opposite angles are equal.

4. If two triangles have two sides of the one equal to two sides of the other, each to each, and also the angles contained by those sides equal, the triangles are congruent.

5. If two sides of a triangle are equal, the angles opposite to these sides are equal.

6. If two angles of a triangle are equal, the sides opposite to these angles are equal.

7. If two triangles have the three sides of the one equal to the three sides of the other, each to each, the triangles are congruent.

Paper 1.

1. (*a*) Put two dots, A and B, 5 centimetres apart.
   (*b*) Join AB.
   (*c*) With centre A and radius 4 cms. draw two small arcs on opposite sides of AB.
   (*d*) With centre B and radius 4 cms. draw two small arcs on opposite sides of AB and cutting the first two arcs.
   (*e*) Mark with very small dots the two points where the arcs intersect and call them P and Q.
   (*f*) Join AP, AQ, BP and BQ

2. In the figure of Question 1 prove that the angle PAB = the angle PBA, and that the angle PAQ = the angle PBQ.

3. In the figure of Question 1 join PQ, and let PQ cut AB in M. Measure
   (*a*) the number of centimetres and of inches in AM,
and (*b*) the number of degrees in each of the angles AQP and BQP.

4. Draw a straight line AB 1·6 inches long, and on AB describe an isosceles triangle ABC, having each of the sides AC and BC double the length of AB.

5. Draw a straight line PQ 7 cms. long, and in PQ take a point A such that PA is 2·8 cms. From A draw a straight line AB making the angle PAB 52°. Measure the angle QAB.

6. In the figure of Question 5 bisect the angle PAB by the straight line AX, and bisect the angle QAB by the straight line AY. Prove that the angle XAY is a right angle, and measure the angle XAY with your protractor in order to see that your drawing is correct.

Paper 2.

1. (*a*) Draw a straight line AB 5·2 cms. long.

(*b*) Bisect AB at M.

(*c*) From M draw MC at right angles to AB.

(*d*) From MC cut off MP 2·6 cms. long.

(*e*) Join AP and BP. Test your drawing by measuring

(i) AM, BP and AP both in inches and in centimetres,

and (ii) each of the angles APM and BPM.

2. In the figure of Question 1, prove that the triangle AMP is equal to the triangle BMP in all respects.

3. Draw an equilateral triangle ABC, and prove that any straight line which bisects one of the angles of the triangle ABC will also bisect the opposite side of the triangle.

4. Draw a straight line AB 2·4 inches long. On AB construct

(*a*) a right-angled isosceles triangle having BAC the right angle;

(*b*) an obtuse-angled isosceles triangle having the angle BAC 120°;

and (*c*) an acute-angled isosceles triangle having BAC 75°. always making AC equal to AB.

5. Draw a straight line PQ, and in PQ take any point A. Through A draw a straight line RAS, cutting PQ and making the angle PAR 54°. Measure each of the angles RAQ, QAS and PAS.

6. In the figure of Question 5 bisect the angles PAR, RAQ, QAS and PAS by the straight lines AX, AY, AZ and AW. Prove that AX and AZ form one straight line, and AY and AW another straight line.

Paper 3.

1. Draw a straight line AB 2·4 inches long. At B erect the perpendicular BC, and from A draw the straight line AC, making the angle BAC 30°, and so completing the triangle ABC. Measure the number of inches and of centimetres in AC and in BC, and the number of degrees in the angle ACB.

2. In the figure of Question 1 bisect the angle ACB by the straight line CD, which meets AB in D. Prove that the angle DCA is equal to the angle DAC and that the triangle ADC is isosceles.

3. Draw two right-angled scalene triangles, equal in all respects. Show by diagrams how two different isosceles triangles, a rectangle, and a kite can all be formed by placing together, in different ways, these two right-angled scalene triangles.

4. Draw a straight line PQ 5·2 cms. long. On PQ construct

(*a*) a right-angled scalene triangle having the angle QPR a right angle;

(*b*) an obtuse-angled scalene triangle having the angle QPR 108°;

and (*c*) an acute-angled scalene triangle having the angle QPR 75°, always making PR equal to half PQ.

5. Draw an isosceles triangle ABC, having the side AB equal to the side AC. Bisect the angle BAC by the straight line AD. Prove that AD will also bisect the base BC.

6. Explain, with examples, the difference between a Problem and a Theorem.

Paper 4.

1. Draw a straight line AB 3·2 inches long. On AB describe a triangle ABC, having the angle CAB a right angle and the angle ABC 30°. Bisect BC at P and join AP. Measure AP and CP in inches and in centimetres, and the number of degrees in each of the angles CAP and BAP.

2. In the figure of Question 1 bisect the angle BAC by the straight line AX, meeting BC in X. Measure XP in centimetres and in inches, and the number of degrees in the angle XAP.

3. Draw a straight line AB 5 cms. long and bisect AB at C; from C draw CD at right angles to AB. In CD take any point E, and join AE and BE. Prove that AE = BE, and verify by measuring AE and BE.

4. A man who is facing S.E. turns round towards the right till he faces N.; through how many degrees will he have turned? If he had turned round towards the left, through how many degrees would he have turned? Draw a figure to illustrate your answers, and show by measurement and by addition that your answers are correct.

5. ABC is an isosceles triangle having the side AB equal to the side AC, and the angle BAC is bisected by the straight line AD. Prove that AD is perpendicular to the base BC, and verify by measuring the angles ADB and ADC.

6 Explain by means of examples the meaning of the complement of an angle, the supplement of an angle, and a reflex angle.

Paper 5.

1. Construct a triangle ABC, having AB 3 inches, BC 4 inches and CA 5 inches long. Measure each of the angles in the triangle ABC.

2. Draw an isosceles triangle ABC having AB = AC, and produce BC both ways to D and E. Prove that the exterior angle ABD is equal to the exterior angle ACE.

3. ABC is any sort of triangle, and the base BC is produced to D. If the adjacent angles ACB and ACD are bisected by the straight lines CP and CQ respectively, prove that CP and CQ are at right angles to each other.

4. A clock is started at 2 o'clock. Through how many degrees will

(*a*) the minute-hand,

and (*b*) the hour-hand

have passed by 3.15?

5. Draw the isosceles triangles ABC, ACD, such that AB = AC = AD = 4·6 cms., and let the triangles have AB and AD in one straight line. Measure the angle BCD.

6. Draw a quadrilateral ABCD, having its opposite sides equal, viz. AB to CD and AD to BC. Join BD. Prove that the angle BAD = the angle BCD.

Paper 6.

1. Construct a right-angled triangle ABC having the angle A 90°, the angle B 60° and the angle C 30°, and the side AB 3·2 cms. long. Measure the sides BC and CA in centimetres and in inches.

2 ABC is an isosceles triangle having AB equal to AC. The angle ABC is bisected by the straight line BD and the angle ACB by the straight line CD, the point D being inside the triangle. Prove that DB = DC.

3. Draw a straight line 2·3 inches long, and on it describe an equilateral triangle.

4. Construct a triangle ABC, having AB = 4·3 cms., BC = 5·4 cms., and CA = 7·6 cms. From A draw AD perpendicular to BC, and measure BD and CD. Also, bisect the angle BAC by the straight line AM, which meets BC in M. Measure BM and MC.

5. ABC and DBC are two isosceles triangles on the same base BC and on the same side of BC, the vertex A being within the triangle DBC. Join AD.

(i) Prove that the angle BDA = the angle CDA.

(ii) What other angles in the figure are equal?

6. Explain, with examples, the meaning of

(*a*) a corollary,

and (*b*) a converse proposition.

SECTION II.

The Exercises and Riders in Sections II. and III., Papers 7 to 18, are based on the same Problems and Theorems as Section I., with the addition of the following:

PROBLEMS.

7. To construct a quadrilateral from simple data.

8. To draw a line through a given point parallel to a given straight line.

THEOREMS.

8. If one side of a triangle is produced, the exterior angle is greater than either of the interior opposite angles.

9. If two sides of a triangle are unequal, the greater side has the greater angle opposite to it.

10. If two angles of a triangle are unequal, the greater angle has the greater side opposite to it.

11. Of all the straight lines that can be drawn to a given straight line from a given point outside it, the perpendicular is the shortest.

12. If two triangles have two angles of the one equal to two angles of the other, each to each, and also one side of the one equal to the corresponding side of the other, the triangles are congruent.

13. If two right-angled triangles have their hypotenuses equal, and one side of the one equal to one side of the other, the triangles are congruent.

14. When a straight line cuts two other straight lines, if

(i) a pair of alternate angles are equal,

or (ii) a pair of corresponding angles are equal,

or (iii) a pair of interior angles on the same side of the cutting line are together equal to two right angles,

then the two straight lines are parallel; and the converse.

15. Straight lines which are parallel to the same straight line are parallel to one another.

16. The sum of the angles of a triangle is equal to two right angles.

17. All the interior angles of any rectilineal figure together with four right angles are equal to twice as many right angles as the figure has sides.

18. If one side of a triangle is produced, the exterior angle is equal to the sum of the two interior opposite angles.

19. If the sides of a convex polygon are produced in order, the sum of the angles so formed is equal to four right angles.

Paper 7.

1. Construct a triangle with sides 2·7, 3·8 and 4·4 inches long respectively. Measure the number of degrees in its largest angle.

2. The straight line which bisects the vertical angle of an isosceles triangle also bisects the base and is perpendicular to it. Prove this.

3. The earth revolves once in 24 hours. What angles does a point on its surface pass through in

(*a*) 2 hours,

(*b*) 2 minutes,

(*c*) 2 seconds?

4. Take any two points A and B, and draw any straight line CD, not passing through either A or B. Join AB and bisect it at E. Through E draw EF at right angles to AB so that EF meets CD in F. Join AF and BF. Prove that AF = BF, and verify by measuring AF and BF in centimetres and in inches.

5. (i) Show by an example that a perpendicular need not be a vertical line.

(ii) Take any point P and any straight line AB. Show how the perpendicular to AB through P may be obtained by folding the paper. Does it matter whether P is on the line AB or not?

6. ABC is any triangle. Prove that the difference between any two sides, BA and AC, is less than the third side BC.

Paper 8.

1. Construct a triangle ABC with sides 52, 33 and 28 millimetres long respectively. Bisect each of its sides at the points X, Y, Z; and at X, Y and Z erect perpendiculars to the sides on which X, Y and Z are situated.

2. If two isosceles triangles are on the same base but on opposite sides of it, prove that the straight line which joins their vertices will bisect their common base and be perpendicular to it.

What difference would it make if the isosceles triangles were both on the same side of the base?

3. ABC is any triangle, and the angle BAC is bisected by the straight line AX, which meets BC in X. Prove that BA is greater than BX. Is CA greater than CX? Give a reason for your answer.

4. Draw a straight line AB 6 inches long; mark C at the middle of AB, and D and E at the points which divide AB into three equal parts. On DE describe a triangle DEF, such that EF and FD are each equal to AD, DE or EB. Join CF and measure its length. Also, measure the number of degrees in the angles ACF and FDE.

5. AB is a given straight line, and C and D are two given points outside the line AB. Find a point X in AB such that CX = DX.

6. Take any point O inside the triangle ABC and join OA, OB, OC. Prove that OA, OB and OC are together greater than half the sum of AB, BC and CA.

Paper 9.

1. Construct a triangle ABC, having AB 2″ and AC 3″ long, and the angle BAC 60°. Construct another triangle PQR, having PQ 2″ and PR 3″ long, but the angle QPR 45°. Find by measurement which triangle has the greater base. What difference must be made in the triangle PQR in order that the base QR may be equal to the base BC?

2. ABCD is a quadrilateral, and its diagonals AC and BD bisect each other at right angles. Prove that ABCD has all its sides equal.

3. Show how to draw a straight line any point in which is equidistant from two given points, A and B.

4. Draw

(*a*) a triangle ABC, with each side 3 cms. long;

(*b*) another triangle DEF, with its sides 3, 3 and 4 cms. long respectively;

and (*c*) a third triangle GHK, with its sides 3, 4 and 5 cms. long respectively.

Write down what you can say about the angles of each of these triangles, and verify your statement by measuring each angle.

5. Draw any triangle ABC; bisect AC in E, join BE, and produce BE to F, making EF equal to BE. Join FC, and prove that the two sides, AB and BC, are together greater than twice the median BE.

6. Draw three parallel straight lines. Draw a straight line making an angle of 52° with one of the parallel lines. Find by measurement the number of degrees in each of the eleven other angles made by this straight line with the three given parallel lines.

Paper 10.

1. Draw a straight line AB 5·4 cms. long, and at the point A, with ruler and compasses only, make an angle of 60°.

2. Draw a quadrilateral ABCD having AB = AD, and BC = CD. Join AC and BD, and prove that AC and BD are at right angles to each other. What is the name of the quadrilateral ABCD?

3. Show how to find a point equidistant from three given points, which are not in the same straight line.

4. Draw a straight line AB 3·5″ long, and bisect AB at C. At A, C and B erect perpendiculars to AB, and in the perpendicular drawn from A take any point P, and in the perpendicular drawn from C take any point Q. Join PQ, and produce PQ to meet the third perpendicular in R. Measure PQ and QR. What can you say about the lengths of AP, CQ and BR?

5. Prove that an isosceles triangle has two of its medians equal

6. A and B are two points on the same side of the line CD. Draw AP perpendicular to CD, and produce AP to E, making PE equal to AP. Join BE, cutting CD in X; and join AX. Prove that AX and BX make equal angles with CD.

Paper 11.

1. Draw a straight line 2·3″ long, and at the point B, with ruler and compasses only, make an angle of 45°.

2. Draw a quadrilateral, of which AD is the longest side and BC the shortest. Prove that the angle ABC is greater than the angle ADC, and the angle BCD greater than the angle BAD.

3. Prove that straight lines which are perpendicular to the same straight line are parallel.

4. Draw a straight line 4·7 centimetres long, and on it describe a triangle having two of its angles 36° and 47° respectively. Measure and calculate the number of degrees in the third angle of the triangle.

5. Prove that if the median AD of the triangle ABC is also perpendicular to the side BC, then ABC is an isosceles triangle.

6. ABC is a triangle having the side BA produced to D, and the exterior angle CAD is bisected by the line AX. If AX is parallel to BC, prove that ABC is an isosceles triangle.

Paper 12.

1. Draw a straight line AB 2·6″ long and on AB describe a triangle, two of whose angles are 60° and 45°, using ruler and compasses only. How many degrees will there be in the third angle of the triangle?

2. ABCD is a quadrilateral, and the two opposite sides AB and CD are bisected at P and Q. Join PQ. If PQ is at right angles to AB and CD, prove that AD = BC.

3. Prove that if a straight line meets two or more parallel straight lines, and is perpendicular to one of them, it is perpendicular to all the others.

4. Draw a triangle PQR, having PQ 4·5 cms., QR 6 cms. and RP 7·5 cms. long. Through P draw a straight line parallel to QR and measure the distance between the parallels.

5. Prove that the three medians of an equilateral triangle are equal. Verify by actual measurement in centimetres and in inches.

6. Take any point O within the triangle ABC and join OA, OB, OC. Prove that OA, OB and OC are together less than the sum of AB, BC and CA.

What else do you know about the lines OA, OB, OC?

SECTION III.

The Exercises and Riders in Section III. are based on the same Problems and Theorems as those in Section II., but are somewhat more difficult.

Paper 13.

1. Draw a straight line AB 2·2 inches long, and at A and B make the angles BAC and ABC each 45°, using ruler and compasses only. From C draw CD perpendicular to AB and measure CD.

2. Draw any triangle ABC and bisect each of the angles ABC and ACB by the straight lines BI and CI. Join IA and prove, by drawing perpendiculars from I to the sides of the triangle, that IA bisects the angle BAC.

3. Prove that any straight line, which is drawn parallel to the base of an isosceles triangle, makes equal angles with the sides, or sides produced, of the triangle.

4. Construct a quadrilateral figure having AB 5 cms., BC 3 cms., CD 6 cms. and AD 1 cm. long, and the angle ABC a right angle. Measure the number of degrees in the angle DAB.

5. ABCD is a quadrilateral having the side AB equal to the side CD, and the angle ABC equal to the angle BCD. Prove by folding over that AD is parallel to BC.

6. From two given points on opposite sides of a given straight line show how to draw two different straight lines which shall meet in the given straight line and make equal angles with it.

Paper 14.

1. Draw a straight line QR 2·7 inches long, and on QR describe a triangle PQR, using ruler and compasses only, having the angle at P 75° and the angle at R 45°.

2. Draw any triangle ABC, and produce the sides AB and AC to D and E. Bisect each of the exterior angles DBC and ECB by the straight lines BI′ and CI′, meeting at the point I′. Join AI′, and prove that AI′ bisects the angle BAC.

3. Prove that the straight line drawn parallel to the base of an isosceles triangle through the vertex will bisect the exterior angles at the vertex.

What would this theorem be if the triangle were equilateral?

4. Draw a straight line BC 46 millimetres long, and on BC describe a quadrilateral having the angle ABC 80°, the angle ACB 40°, the angle DBC 35° and the angle DCB 70°. Measure AD in inches and in millimetres.

5. Prove that if two circles cut one another, the straight line joining their points of intersection is bisected at right angles by the line joining the centres of the circles.

6. AX is a given finite straight line, and P and Q are two given acute angles. Show how to construct a triangle ABC, having the angle ABC equal to P, the angle ACB equal to Q, and AX the perpendicular from A to BC.

Paper 15.

1. Construct a quadrilateral ABCD, having AB 3·3 cms., BC 4 cms., CD 2·5 cms., DA 5 cms. long, and the angle BAD 45°.

Find also graphically the greatest and the least possible values of the angle BAD.

2. What can be said about the remaining angles of a triangle if we know that

(*a*) one angle of the triangle is a right angle,

or (*b*) the triangle is isosceles and contains a right angle,

or (*c*) the triangle is isosceles and one of its exterior angles is 75°?

3. AB is a given straight line and P is a given point. Show how to draw through P straight lines making an angle of 51° with AB. How many such straight lines can be drawn? What difference does it make whether P is on or not on the line AB?

4. Construct a right-angled triangle having the side AB 3·5 inches long and the angle BAC 52°. Show that this can be done in two and only two ways.

5. If in a quadrilateral two opposite sides be equal, and the angles which a third side makes with the equal sides be equal, prove that the other angles of the quadrilateral must be equal.

6. Draw an obtuse-angled triangle ABC, and bisect the obtuse angle at A by the straight line AX. From C draw CPQ perpendicular to AX to meet AX in P and AB in Q. Prove that each of the angles AQC and ACQ is equal to half the sum of the angles ABC and ACB.

Paper 16.

1. Construct a quadrilateral ABCD, having AD and BD each 4·8 cms. long, BC 3 cms., the angle CBD 25°, and the angle ABD 30°. Measure AC in centimetres and in inches.

2. Prove that the sum of the angles of any quadrilateral figure is equal to four right angles.

3. Show that straight lines which make equal angles with two given intersecting straight lines form two sets of parallel straight lines.

4. Trisect a right angle, using ruler and compasses only.

5. Find all the points which are distant 4 cms. from one, and 3 cms. from the other, of two given intersecting straight lines.

6. A quadrilateral ABCD has its sides AB 2·8 cms., BC 2·5 cms., CD 1·7 cms. and DA 3 cms. long. What is the greatest possible length for either of the diagonals AC or BD?

Paper 17.

1. Draw a straight line AB 9·4 cms. long, and on AB describe a triangle ABC having the angle ABC three times as large as the angle BAC, and five times as large as the angle ACB.

2. Prove that if a straight line is divided into any two unequal parts, the distance of the point of section from the middle of the line is equal to half the difference of the two unequal parts.

3. Prove that straight lines which make a given acute angle with a given straight line form two sets of parallel lines.

4. Construct a quadrilateral ABCD, having AB = BC = 2″, CD = 1″, and each of the angles ABC and BCD 108°. How many degrees are there in the angle DAB?

5. Find all the points equidistant from the three sides, or sides produced, of a given triangle.

6. BAC and BDC are two triangles on the same base BC, and on the same side of it, and the angle BAC is equal to the angle BDC. Prove that each of the vertices A and D must lie without the other triangle.

Paper 18.

1. Draw a triangle ABC in which AB = 3·6″, AC = 2·7″, and the angle BAC is 52°. Find the two points P and Q each of which is equidistant from BA and CA, produced if necessary, and each of which is also equidistant from B and C. Measure PQ in centimetres and in inches.

2. Prove that, if the straight line AB is bisected at C and produced to D, then CD is equal to half the sum of AD and BD.

3. Two unequal straight lines AB and CD bisect each other at the point P. Join AC and BD, and prove that AC and BD are equal and parallel.

4. Draw a straight line BC 5·6 cms. long, and on BC describe a triangle ABC, having the angle ABC 50° and the angle ACB 65°. Bisect AB at D, and draw DE parallel to BC, cutting AC at E. Measure AE, EC and DE.

5. Given two points A and B on the same side of a straight line CD. Find a point P in CD, such that the sum of AP and BP is a minimum.

6. ABC is any triangle. Bisect AB in P, and through P draw any straight line QPR meeting CB in Q, and CA produced in R. From PR cut off PN equal to PQ, and join AN. Prove that the triangle APN is equal to the triangle PQB, and that the triangle ABC is less than the triangle QRC.

SECTION IV.

The Exercises and Riders in Sections IV. and V., Papers 19 to 30, are based on the same Problems and Theorems as the previous Sections, with the addition of the following:

PROBLEM.

9. To divide a straight line into a given number of equal parts or into parts in any given proportion.

THEOREMS.

20. The opposite sides and angles of a parallelogram are equal, and each diagonal bisects the parallelogram.

21. If there are three or more parallel straight lines, and the intercepts made by them on any straight line that cuts them are equal, then the corresponding intercepts on any other straight line that cuts them are also equal.

22. Parallelograms on the same or equal bases and of the same altitude are equal in area.

23. Triangles on the same or equal bases and of the same altitude are equal in area.

24. Equal triangles on the same or equal bases are of the same altitude.

Paper 19.

1. Construct a triangle ABC, having AB = 7·5 cms., $\angle A = 37°$ and $\angle B = 53°$. Bisect AB at M, and join CM. Measure CM in centimetres and in inches.

2. Show that four equal right-angled isosceles triangles can be arranged round one common vertex so as to form a square.

3. In the triangle ABC the side AB is bisected at M, and MN is drawn, parallel to BC, to meet AC in N. Prove, by drawing NP parallel to AB, that MN bisects the side AC.

4. Describe a quadrilateral which shall have two sides parallel and one inch apart, and the other two sides equal, but not parallel, and 2 inches long.

5. A triangle is divided by each of its medians into two triangles of equal area.

6. The diagonals of a parallelogram bisect each other.

Paper 20.

1. Construct a triangle ABC, having AB = 6·9 cms., BC = 7·5 cms. and CA = 7·8 cms. Bisect AB and AC at M and N, and join MN. Measure MN in centimetres and in inches.

2. Show that six equilateral triangles can be arranged round one common vertex so as to form a regular hexagon.

3. The straight line which joins the middle points of two sides of a triangle is parallel to the third side.

4. Draw a plan of a cross vertical section of a ditch whose depth is 3 feet, breadth at the bottom 2 feet, and at the top 4 feet, the sides being equal. Represent 1 foot by 1 inch, and find the length of the sides of the ditch.

5. Given the area and the base of a triangle, find the locus of its vertex.

6. The four triangles into which a parallelogram is divided by its diagonals are equal in area.

Paper 21.

1. Draw a straight line 9·8 cms. long, and divide it into seven equal parts. Verify your work by measuring each part.

2. Prove that there is one and only one point equidistant from three given points not in the same straight line.

3. Show how to bisect a triangle by a straight line drawn through one of its angular points. When will this straight line also bisect the angle of the triangle?

4. Construct a rectangle whose sides are 4·3″ and 2·1″ long respectively. Measure the angles which a diagonal of this rectangle makes with two opposite sides.

5. Draw an isosceles triangle ABC, having each of the equal sides, AB and AC, twice the length of the base BC. In BC take any point P, and draw PM and PN parallel to AC and AB respectively, to meet AB in M, and AC in N. Prove that PM+PN is constant. Prove, also, that this is true for all isosceles triangles.

6. ABCD is a parallelogram, and PAB, QBC, RCD, SDA are equilateral triangles described outside the parallelogram on the bases AB, BC, CD, DA. Join PQ, QR, RS, SP, and prove that the opposite sides SP and QR are equal and parallel.

Paper 22.

1. Draw a straight line AB 3″ long, and suppose it represents 2 feet 6 inches. Produce AB, and mark off the length that will represent 3 feet 4 inches.

2. If any point O is taken on the median AM of the triangle ABC, prove that the triangle AOB is equal to the triangle AOC in area.

3. Show how to bisect a triangle by a straight line drawn through any given point in one side of the triangle. Explain how this method of bisecting a triangle becomes simplified when the given point is the middle of the side.

4. Construct a rhombus having its sides 4·7 cms. long, and containing an angle of 60°, using ruler and compasses only.

5. Prove that the area of a parallelogram is equal to the product of the base into the altitude, and deduce a rule for finding the area of a triangle.

6. Any straight line AX is drawn through the angle A of the parallelogram ABCD, and BP, CQ and DR are drawn perpendicular to AX. If C is the angle opposite A, prove that CQ is equal to the sum or difference of BP and DR, according as AX falls without or within the parallelogram.

SECTION V.

The Exercises and Riders in Section V. are based on the same Problems and Theorems as those in Section IV., but are somewhat more difficult.

Paper 25.

1. Construct a triangle ABC having the $\angle B = 45°$, $AB = 3{\cdot}4$ cms. and $AC = 2{\cdot}8$ cms. Show that two such triangles can be drawn, and measure BC in inches and in centimetres in each case.

2. Prove that the straight lines which bisect two opposite angles of a parallelogram are either parallel or coincident. When are the bisectors coincident?

3. The three straight lines which join the middle points of the sides of any triangle divide the triangle into four triangles which are equal in all respects.

4. Construct an irregular polygon having $AB = 3{\cdot}25''$, $BC = 2{\cdot}75''$, $CD = 3{\cdot}5''$, $DE = 3{\cdot}75''$, $\angle B = 110°$, $\angle C = 100°$ and $\angle D = 105°$. Measure the two remaining angles of the polygon.

5. A regular rectilinear figure has each of its angles 108°. How many sides has it? Prove that your answer is correct.

6. OA and OB are two given intersecting lines meeting in O, and P is a given point not in either OA or OB. Show how to draw through P a straight line PQ meeting OA, or AO produced, in Q, so that PQ is bisected by OB. When must AO be produced?

Paper 26.

1. On a given base AB, 5 cms. long, construct an isosceles triangle ABC whose area is 6·25 square centimetres. On the same base AB construct another isosceles triangle whose area is twice the area of ABC. Measure the vertical angles of each isosceles triangle.

2. Prove that straight lines which bisect two adjacent angles of a parallelogram intersect at right angles.

3. The three medians of any triangle are concurrent, and cut one another in a point such that the smaller part of any median is one-third of the whole median.

4. Construct an irregular polygon ABCDEF having the sides AB = 3·76″, BC = 3·26″, CD = 2·1″, DE = 2·24″, AF = 2·18″, and the diagonals AC = 5·36″, AD = 5·14″, AE = 4·52″ and BF = 5·2″. Measure the angles ABC and BCD.

5. A regular rectilinear figure has each of its angles 150°. How many sides has it? Prove that your answer is correct.

6. AB and AC are two straight lines, and P is a given point not in either AB or AC. Show how to draw through P a straight line meeting AB in D and AC in E, so that ADE is an isosceles triangle.

Paper 27.

1. Construct a parallelogram having two of its sides 7 cms. and 5 cms., and one of its angles 60°, without using a protractor. Divide the parallelogram into 3 equal parts by lines drawn through one of its corners.

2. Prove that every straight line drawn through the point of intersection of the diagonals of a parallelogram bisects the area of the parallelogram.

3. ABCD is an irregular quadrilateral, and X, Y, Z, W are the middle points of its sides. Prove that the figure formed by joining XY, YZ, ZW and WX is a parallelogram.

4. If one angle of a triangle is 20° greater than the second angle, and if the third angle is a right angle, find the number of degrees in the second angle. Draw the triangle, having the hypotenuse 3·6″ long, and measure the shortest side of the triangle in inches and in centimetres.

5. The bisectors of the angles of a triangle are concurrent.

6. If ABC and ABD are two triangles, equal in area, on the same base AB, but on opposite sides of AB, prove that the triangles have equal altitudes, and that the straight line joining the vertices C and D is bisected by AB.

Paper 28.

1. Construct a parallelogram having two of its sides 5″ and 3″, and one of its angles 52°. Divide the parallelogram into 5 equal parts by lines drawn through one of its corners.

2. If two opposite sides of a quadrilateral are parallel, the straight line which joins the middle points of these two parallel sides will bisect the area of the quadrilateral.

3. In any triangle ABC the angle contained by the bisector of the vertical angle A, and the perpendicular drawn from the vertex A to the base BC, is equal to half the difference of the angles B and C at the base of the triangle.

4. On a given base 1·5″ long construct an isosceles triangle having each of the angles at the base double the angle at the vertex.

5. The straight line drawn through the middle point of one side of a triangle, parallel to another side, will bisect the third side of the triangle.

6. Prove that two quadrilaterals are equal in area, if their diagonals are equal and if the diagonals intersect each other at equal angles.

Paper 29.

1. Draw a straight line 5·6 cms. long, and on it describe a square ABCD. In the square inscribe an equilateral triangle APQ, having the point P in BC and the point Q in CD. Measure the length of AP in centimetres and in inches.

2. ABC is any triangle having the side BC produced to D. Bisect the angles ABC and ACD by the lines BP and CP meeting in P. Prove that the angle BPC is equal to half the angle BAC.

3. Show that every right-angled triangle can be divided into two isosceles triangles by drawing a straight line from the right angle to a certain point in the hypotenuse. What can you say about the length of the straight line so drawn?

4. Suppose that you have taken the measurements of the sides of a triangular field, namely, 15 chains, 20 chains and 25 chains. Draw a plan of the field, using a scale of 10 chains to 1 inch and find the area of the field in square chains.

5. ABC is any given triangle. Show how to describe another triangle PQR, having the points A, B, C as middle points of its sides. Hence show that the three straight lines drawn from the angular points of the triangle ABC perpendicular to its opposite sides are concurrent.

6. ABC is any triangle and D is any point in AB. Show how to draw through D a straight line DE to meet BC produced in E, so that the triangle DBE may be equal to the triangle ABC in area

Paper 30.

1. Construct a quadrilateral ABCD, having AB = 3·2 cms., BC = 2·1 cms., CD = 3 cms., DA = 4·1 cms., and ∠ B = 30°, without using a protractor. Construct a triangle equal in area to ABCD, and so find its area. Verify by drawing the quadrilateral on squared paper and counting squares.

2. Find the locus of the middle points of all straight lines drawn from a fixed point to meet a given straight line of unlimited length.

3. A number of right-angled triangles have a common right angle and equal hypotenuses. Prove that the middle points of the hypotenuses all lie on the circumference of a circle.

4. On a given base AB, 2″ long, construct a triangle ABC, whose area is 2·9 square inches, and one of whose angles is 45°.

5. If the angular points of one parallelogram lie on the sides of another parallelogram, the diagonals of both parallelograms pass through the same point.

6. Find in a side of a triangle the point from which the straight lines, drawn parallel to the other sides of the triangle and terminated by them, are equal.

SECTION VI.

The Exercises and Riders in Sections VI. and VII., Papers 31-42, are based on the same Problems and Theorems as the previous Sections, with the addition of the following :

PROBLEM.

10. To construct a triangle equal in area to a given polygon.

THEOREMS.

25. In a right-angled triangle the square described on the hypotenuse is equal to the sum of the squares described on the other two sides of the triangle.

26. The locus of a point which is equidistant from two fixed points is the perpendicular bisector of the straight line joining the two fixed points.

27. The locus of a point which is equidistant from two intersecting straight lines consists of the pair of straight lines which bisect the angles between the two given lines.

Questions are also set on simple Graphs and on the use of Squared Paper.

Paper 31.

1. Draw on squared paper a rectangle whose length is 11·9 cms. and breadth 7·7 cms. Calculate the area of the rectangle, and verify your result by counting the number of squares inside the rectangle.

2. A given straight line AB is bisected at C, and AX, CZ, BY are drawn perpendicular to any other straight line. Prove that the projections of AC and CB, that is, XZ and ZY, are equal.

3. In a rhombus the squares described on the four sides are together equal to the squares described on the diagonals.

4. Construct a parallelogram ABCD, whose area is 9 square centimetres, and whose sides, AB and AD, are 6 and 3 centimetres long respectively.

5. Prove that if the angle DAB between the adjacent sides of a parallelogram ABCD is increased, but the lengths of the sides AD and AB remain the same, then the length of the diagonal AC will be decreased.

6. ABC is a triangle obtuse-angled at C. AC and BC are bisected at D and E, and DF and EG are drawn at right angles to AC and BC, meeting AB in F and G. Join CF and CG. Prove that the perimeter of CFG = AB, and that the angles CFG and CGF are double of the angles CAB and CBA respectively.

Paper 32.

1. Construct a triangle ABC, having AB = 1″, BC = 2·4″ and CA = 2·6″, and prove that one angle of the triangle is a right angle.

2. If the diagonals of a quadrilateral divide it into four triangles equal in area, prove that the quadrilateral is a parallelogram.

3. ABCD is a parallelogram. Prove that the diagonals EG, HK, of the parallelograms AEFG and FHCK about the diagonal AC, are parallel.

4. Construct a right-angled isosceles triangle ABC, having each of the equal sides AB and AC 3·6 cms. long. Calculate the length of the base BC and verify your result by measurement.

5. When the two sides AB, AC of a triangle are given in length, the area of the triangle is a maximum when BAC is a right angle.

6. Draw any two straight lines AB and AC containing any angle BAC. From AB cut off AD = DE = EF, and from AC cut off AG = GH = HK. Join DG, EH, FK and prove that DG, EH and FK are parallel.

Paper 33.

1. Draw a straight line AB and in it take a point P. Construct a triangle PQR having PQ = 5·6 cms., PR = 7·2 cms., QR = 3·1 cms., and QR parallel to AB. Measure each of the angles in the triangle PQR.

2. The area of the square described on AC, a diagonal of a given square ABCD, is double the area of the square ABCD.

3. Prove that the straight lines which join the extremities of two equal and parallel straight lines either bisect each other or are equal and parallel. Distinguish between the two cases.

4. Describe a square whose area is 5 square inches without actually calculating the square root of 5.

5. Prove that the three medians of any triangle are always less than the three sides of the triangle.

6. Describe a triangle having an angle of 52°, and equal in area to a given parallelogram. Prove your construction.

Paper 34.

1. Construct a trapezium ABCD, having AB = 3·1″, BC = 4·2″, CD = 5·6″, DA = 3·4″ and AD parallel to BC. Measure each of the angles in the trapezium.

2. ABC is an equilateral triangle, and AD is drawn perpendicular to BC. Prove that the square on AD is equal to three times the square on BD or CD.

3. AB and AC are two intersecting straight lines. Show how to describe an isosceles triangle having its base on AB and its vertex on AC, each of the equal sides being double of the base.

4. A man walks 1 mile due W., then he turns 35° to his left and walks 2 miles, then he turns 45° to his right and walks 1 mile, and then turns N.E. and walks 3 miles. Plot his path on squared paper and find

(*a*) what angle he turns through altogether,

and (*b*) how far he is away from his starting point.

5. Equal and parallel straight lines have equal pro jections on any other straight line.

6. M is the middle point of the side BC of the triangle ABC, and AM is bisected in P. If BP produced meets AC in F, then $AF = \frac{1}{3}AC$.

Paper 35.

1. Plot the following four points: (*a*) 5, 7; (*b*) −8, 2; (*c*) −7, −6; (*d*) 3, −5. Draw the six lines joining each pair of these points and find the co-ordinates of the middle point of each of the six lines.

2. Take any triangle OEC and draw its median CX. Produce CX to B, making XB = CX, and produce EO to A, making OA = EO. Join AB and AC, and prove that ABC will be a triangle having its medians equal to $1\frac{1}{2}$ times the various sides of the triangle OEC.

3. ABC is any triangle, and from any point O, perpendiculars OX, OY, OZ are drawn to the sides BC, CA, AB respectively. Prove that

$$AZ^2 + BX^2 + CY^2 = AY^2 + CX^2 + BZ^2.$$

4. Construct a triangle ABC, having AB = 1·2″, BC = 2″ and CA = 1·7″. Then construct another triangle BLM, equal in area to the triangle ABC and having the same angle at B, but having BL = ·8″.

5. In a certain pentagon one of the angles is 120°; of the other four angles one is double, another three times, and another four times the smallest angle. Find the number of degrees in each angle of the pentagon and draw such a pentagon.

6. ABCD is a trapezium having AD parallel to BC. Prove that the straight line MN, which joins the middle points of AB and CD, $= \frac{1}{2}(AD + BC)$; and that the part of MN, which is intercepted between the diagonals AC and BD, $= \frac{1}{2}(AD \sim BC)$.

Paper 36.

1. Plot the following points $(-2, 1)$, $(6, 1)$, $(2, 9)$. Show that they are the angular points of an isosceles triangle. Calculate and measure the lengths of the two equal sides.

2. Show how to construct a triangle having its medians equal to three given straight lines.

3. In the triangles ABC, DEF, AB is equal and parallel to DE, and AC is equal and parallel to DF; prove that BC is equal and parallel to EF.

4. Construct a square ABCD, with side 2″. Draw the diagonal AC and produce AC to E, making CE = 2″. Join BE, and draw EF perpendicular to AB produced. Calculate and measure

(*a*) the lengths of AE, BE, EF to the nearest hundredth of an inch,

and (*b*) the size of the angles AEB, ABE.

5. ABC is any triangle, and the angle BAC is bisected by the straight line AO, which meets in the point O the straight line which bisects the base BC at right angles. From O draw OM, ON, perpendiculars to AB, AC, meeting AB, AC in M and N. Prove that AM = AN and BM = NC.

6. If the opposite angles of a quadrilateral are supplementary, a point can be found which is equidistant from its four vertices.

SECTION VII.

The Exercises and Riders in Section VII. are based on the same Problems and Theorems as those in Section VI., but are somewhat more difficult.

Paper 37.

1. A certain polygon has 7 sides, and one of its angles is 25° greater than each of the other angles, which are all equal; find the number of degrees in each angle. If the five sides which join vertices at which the angles are equal are each 1″, construct the polygon and measure the remaining sides.

2. Draw any triangle ABC and bisect each of the angles ABC and ACB by the straight lines BI and CI meeting in I. Prove that the angle BIC is 90° greater than half the angle BAC.

3. O is any point outside the parallelogram ABCD and also outside the angle BAD and its opposite vertical angle. Prove that the triangle OAC will be equal in area to the sum of the areas of the triangles OAD, OAB.

4. Plot the following points and show that they all lie on a circle whose centre is the origin: (*a*) 6, 8; (*b*) 0, 10; (*c*) −6, 8; (*d*) −6, −8; (*e*) −10, 0.

5. ABC is a triangle having the angle BAC a right angle, and AD is drawn perpendicular to BC and meeting BC in D. Prove, by using squared paper, that the square on BA is equal to the rectangle contained by BC and BD.

6. Take any triangle ABC and from B draw BP perpendicular to the bisector of the vertical angle BAC and meeting the bisector in P. Bisect the base BC in M, and join PM. Prove that $PM = \frac{1}{2}(AB - AC)$.

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Paper 38.

1. Two of the angles of a certain octagon are right angles, two other angles are each $1\frac{1}{2}$ right angles; and of the four remaining angles one is 10° greater, and another 20° less than each of the other two angles. Find the number of degrees in each angle and draw such an octagon.

2. A straight line drawn from the vertex of a triangle to any point in the base is bisected by the straight line which joins the middle points of the other sides of the triangle.

3. ABCD is a parallelogram, and O is a point within the angle BAD or its opposite vertical angle. Prove that the triangle OAC is equal in area to the difference of the areas of the triangles OAD, OAB.

4. Plot the following points and show that they are the angular points of a rhombus: (2, 9), (15, 9), (10, −3), (−3, −3).

5. P is any point in the base BC of the isosceles triangle ABC. BP and PC are bisected at M and N, and from M and N lines are drawn at right angles to BC, meeting the sides, AB in H and AC in K. Prove that BH = AK.

6. BP, CQ are perpendiculars from the ends of the base BC of a triangle ABC upon the bisector of the vertical angle A, and M is the middle point of the base BC. Prove that MP = MQ.

Paper 39.

1. Construct on squared paper a polygon ABCDEF, having AB = 1″, BC = 1·2″, CD = 1·4″, DE = 1·6″, EF = 1·8″, ∠ ABC = 140°, ∠ BCD = 130°, ∠ CDE = 120° and ∠ DEF = 110°. Measure the side FA in inches and in centimetres, and show that BC is parallel to EF.

2. In a triangle ABC, AD is drawn perpendicular to BC, and X, Y, Z are the middle points of the sides BC, CA, AB. Prove that each of the angles ZXY, and ZDY is equal to the angle BAC.

3. A, B, C, D are four towns, which, if joined in this order, would form a convex quadrilateral. Prove that, to get from A to D via B and C, ABCD is always a shorter route than ACBD.

4. In the triangle ABC the angle BAC is 90°, and the side AB = 3 cms. Plot a graph to show how BC varies for different values of AC. Use the graph to find the value of AC when BC = 4 cms., and verify the result by drawing this triangle.

5. Show how to divide a given straight line into two parts, so that the square on one part may be double the square on the other part.

6. Prove that the straight lines which join the middle points of the sides of any quadrilateral form a parallelogram whose area is equal to half the area of the quadrilateral.

Paper 40.

1. Construct on squared paper a polygon ABCDEF, having AB = 4 cms., AC = 6·5 cms, AD = 8 cms., AE = 6·5 cms., AF = 3·5 cms., ∠ BAC = 35°, ∠ CAD = 25°, ∠ DAE = 40° and ∠ EAF = 25°. Find the area of the polygon by reducing it to a triangle, and verify your result by counting the squares in the polygon.

2. O is a point inside the triangle ABC, such that the triangles AOB, AOC are equal in area. Prove that AO produced will bisect BC.

3. If one triangle lies entirely inside another triangle, prove that the perimeter of the former triangle is always less than the perimeter of the latter triangle.

4 Plot the following points and find the area of the triangle formed by joining the points: (5, 3), (−12, 1), (−6, −4).

5. Show how to divide a given straight line into two parts, so that the square on one part may be three times the square on the other part.

6. The quadrilateral, which is formed by the four straight lines bisecting the angles of any quadrilateral, has its opposite angles equal to two right angles.

Paper 41.

1. Draw a straight line whose length is $\sqrt{13}$ cms. without actually calculating the value of the square root of 13.

2. The perpendiculars through the middle points of the sides of the triangle ABC meet in O, and the perpendiculars to the sides of the triangle from the opposite angles of the triangle meet in P. Prove that AP is equal to twice the length of the perpendicular from O to BC.

3. Prove that each of the angles of a regular polygon is equal to the angle between the alternate sides of a polygon having twice the number of sides as the first polygon.

4. Plot the following points, show that they are the angular points of a trapezium, and find the area of the trapezium: (5, 1), (1, 1), (8, −4), (0, −4).

5. Having given the direction of two lines AB and AC, and that BC always passes through a fixed point P, prove that the area of the triangle ABC will be a minimum when BC is bisected at P.

6. The quadrilateral, which is formed by the four straight lines bisecting the exterior angles of any quadrilateral, has its opposite angles supplementary.

Paper 42.

1. Construct on squared paper a parallelogram ABCD, having AB = 3·5 cms., BC = 4 cms. and AC = 4·5 cms. On BC describe an isosceles triangle having the same area as the parallelogram. Find this area by measurement and verify your result by counting squares.

2. The point of concurrence of the perpendiculars to the sides of a triangle at their middle points, the point of concurrence of the perpendiculars to the sides from the opposite angles, and the point of concurrence of the medians are collinear.

3. Having given two lines AB and CD not parallel to each other, find, without producing AB and CD, the straight line which, if produced, would bisect the angle that would be formed between AB and CD when produced to meet.

4. Plot the following points, show that they are the angular points of a parallelogram, and find its area: (4, 6), (7, 2), (– 5, – 3), (– 8, 1).

5. ABC is any triangle and P is any point in its base AB. From P draw PQ and PR parallel to the sides CB and CA of the triangle and meeting the sides in Q and R. Prove that the parallelogram PQCR is greatest when the point P is taken at the middle point of AB.

6. The triangle formed by the three bisectors of the exterior angles of a triangle is such that the lines joining its vertices to the angles of the original triangle will be its perpendiculars.

SECTION VIII.

The Exercises and Riders in Section VIII., Papers 43 to 48, are based on the same Problems and Theorems as the previous Sections, with the addition of the following:

PROBLEMS.

11. To divide a given finite straight line in medial section.

12. To construct a square equal in area to a given polygon.

THEOREMS.

28. Illustrations and explanations of the geometrical theorems corresponding to the following algebraical identities:

(i) $\mathsf{K}(a+b+c+\ldots) = \mathsf{K}a + \mathsf{K}b + \mathsf{K}c + \ldots,$

(ii) $(a+b)^2 = a^2+2ab+b^2,$

(iii) $(a-b)^2 = a^2-2ab+b^2,$

(iv) $(a+b)\,(a-b) = a^2-b^2.$

29. The square on a side of a triangle is greater than, equal to, or less than the sum of the squares on the other two sides, according as the angle contained by those sides is obtuse, right, or acute. The difference in the case of inequality is twice the rectangle contained by one of the two sides and the projection on it of the other.

Paper 43.

1. Construct a rectangle with one side 3 cms. long, and a diagonal 5 cms. Measure the length of the other side of the rectangle.

2. Construct a quadrilateral ABCD, having each of the sides AB, BC, CD equal to 1·6″, and each of the angles BAD, CDA equal to 70°. If AC, BD intersect in O, prove that BO = CO.

3. Show that the square on any given straight line is equal to four times the square on half the line.

4. Construct a square ABCD, having each side 2·3″ long. In the sides AB, BC, CD, DA, take points P, Q, R, S, such that AP = BQ = CR = DS = ·7″. Measure the sides and angles of the figure PQRS; and hence prove that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides.

5. The sides of a triangle are 10, 12, 15 millimetres. Prove that the triangle is acute-angled.

6. If a line AB is divided equally at C, and unequally at D, prove that $AD^2 \sim DB^2 = 2AB \cdot CD$.

Paper 44.

1. On squared paper construct a rhombus, having its sides 3·6 cms. long and one of its angles 45°. Calculate the area of the rhombus, and verify by counting squares.

2. Prove that, if a straight line is divided into two parts, the rectangle contained by the parts is a maximum, and the sum of the squares on the two parts is a minimum, when the two parts are equal.

3. ABC is an isosceles triangle, and D is any point in its base BC. Prove that AD^2 is less than AB^2 by twice the rectangle BD . DC.

4. Draw on squared paper a plan of a field which is in the shape of a trapezium, the parallel sides being 40 and 56 yards long, and the other sides 22 and 28 yards. Calculate the area of this field, and verify by counting squares.

5. AB is a straight line divided in medial section, so that $AE^2 = AB \,.\, EB$. From AE mark off a part equal to EB, and prove that AE is divided in medial section.

6. If any four points A, B, C, D are taken in order along a straight line, prove that $AB \,.\, CD + BC \,.\, AD = AC \,.\, BD$, and that this is the same as $AB \,.\, CD + BC \,.\, AD + CA \,.\, BD = 0$.

Paper 45.

1. A boy 4 ft. 6 in. high stands at a distance of 8 ft. from a lamp-post, and his shadow is 8 ft. long. Draw a diagram to scale and find the height of the lamp-post.

2. In a right-angled triangle a perpendicular is drawn from the right angle to the hypotenuse. Prove that the square on the perpendicular is equal to the rectangle contained by the segments of the base.

3. A and B are two fixed points, and the point D moves so that $AD^2 \sim BD^2 =$ constant. Prove that the locus of D is a straight line perpendicular to the line AB.

4. Draw a parallelogram, having sides 70 and 80 mms. long, and containing an angle of 128°. Measure the two altitudes of this parallelogram.

5. AB is a straight line divided in medial section, so that $AE^2 = AB \,.\, EB$. Show that the rectangle contained by the sum and difference of the parts AE and EB is equal to the rectangle contained by the parts AE, EB.

6. State and prove geometrically that

$$a(b-c)+b(c-a)+c(a-b)=0.$$

Paper 46.

1. Draw a plan of a four-sided field PQRS, in which the angle QRS = 90°, the side RS = 48 yds., QR = 64 yds. and PQ = PS = 96 yds. Find the area of the field to the nearest square yard.

2. In a right-angled triangle a perpendicular is drawn from the right angle to the hypotenuse. Prove that the square on either of the sides containing the right angle is equal to the rectangle contained by the hypotenuse and the segment of it adjacent to that side.

3. Prove that of all triangles on a given base and with a given area, the isosceles has the least perimeter.

4. Construct a rhombus, having each side 2·4″ long and containing an angle of 105°. Inscribe a square in the rhombus and measure a side of the square.

5. In any triangle the sum of the squares on two sides is equal to twice the square on half the third side, together with twice the square on the median which bisects the third side.

6. State and illustrate geometrically that

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca.$$

Paper 47.

1. Show how to draw a straight line $\sqrt{7}$ inches long, and hence find the value of $\sqrt{7}$ correct to two decimal places.

2. Show how to divide a given straight line into two parts, so that the difference of the squares on the two parts may be equal to a given square. When does the construction appear to be impossible?

3. Prove that of all rectangles having the same perimeter the square has the greatest area.

4. On the axis of co-ordinates OX take a point S 2 cms. from the origin O. Plot the locus of a point P, which moves so that its distance from the axis OY is always equal to its distance from the fixed point S.

5. Prove that three times the sum of the squares on the sides of any triangle is equal to four times the sum of the squares on its medians.

6. Prove that the difference of the squares on AB, AC, two sides of a triangle ABC, is equal to twice the rectangle contained by the base BC and that part of the base intercepted between the middle point of BC and the foot of the perpendicular AD from the opposite angle A to BC.

Paper 48.

1. Draw a triangle ABC having $a = 2{\cdot}4''$, $b = 3{\cdot}2''$ and $c = 3{\cdot}6''$. Draw any straight line parallel to BC, meeting AB in D and AC in E. Join BE and CD, meeting in R. Join AR and produce AR to meet BC in S. Measure BS and SC in inches and in centimetres.

2. Prove that in any quadrilateral the sum of the squares on the two diagonals is equal to twice the sum of the squares on the four straight lines joining the middle points of adjacent sides of the quadrilateral.

3. The sum of the squares of the distances of a point D from two given points A and B is constant. Prove that the locus of D is a circle, whose centre is the middle point of the line joining A and B.

4. Take two points S and H $2{\cdot}4''$ apart, and plot the locus of a point P, which moves so that $SP + HP = 3{\cdot}6''$.

5. Show how to produce a given straight line so that the rectangle contained by the whole line thus produced and the part produced may be equal to the square of the original line.

6. Prove that the sum of the squares on the four sides of any quadrilateral is equal to the sum of the squares on its diagonals plus four times the square on the line joining the middle points of the diagonals.

What does this theorem become when the quadrilateral is a parallelogram?

SECTION IX.

The Exercises and Riders in Sections IX. and X., Papers 49 to 60, are based on the same Problems and Theorems as the previous Sections, with the addition of the following:

PROBLEMS.

13. To construct tangents to a given circle.

14. To construct (*a*) the inscribed, (*b*) the escribed, and (*c*) the circumscribed circles of a given triangle.

THEOREMS.

30. A straight line, drawn from the centre of a circle to bisect a chord which is not a diameter, is at right angles to the chord; conversely, the perpendicular to a chord from the centre bisects the chord.

31. There is one circle, and one only, which passes through three given points not in a straight line.

32. Equal chords of a circle are equidistant from the centre; and the converse.

33. The tangent at any point of a circle and the radius through the point are perpendicular to one another.

34. If two circles touch, the point of contact lies on the straight line through their centres.

Paper 49.

1. Draw two circles to touch each other externally, one with radius 2·8″ and the other with radius 1·8″. Mark very lightly the point where the circles touch. Join the centres and measure the distance between the centres in inches and in centimetres.

2. ABC is a triangle, and the perpendicular drawn from B to AC bisects AC. Prove that BA = BC.

3. Find the locus of the centres of all circles which pass through two given points.

4. Draw a circle with radius 3·8 cms., and draw any diameter of the circle. Take any three points A, B, C on the circumference and all on the same side of the diameter. Mark the three points corresponding to A, B, C on the other half of the circumference, and measure in centimetres and in inches the distances of all these points from the diameter.

5. ABCD is a convex quadrilateral, and P is any point within ABCD. Prove that PA, PB, PC, PD are together greater than half the perimeter of ABCD. Verify by actual measurement.

6. Find the locus of the middle points of any number of parallel chords in a circle.

Paper 50.

1. Draw two circles to touch each other internally, one with radius 3·4 cms. and the other with radius 2·1 cms. Join the point of contact to the centre of the larger circle, and measure the distance between the centres in centimetres and in inches.

2. If in a triangle the straight lines drawn from the extremities of the base perpendicular to the opposite sides are equal, the triangle is isosceles.

3. Find the locus of the centres of all circles which touch a given straight line at a given point.

4. Describe a circle ABC with radius 1·2 inches long. On the circumference of this circle mark any point P, and then describe two different circles to touch the circle ABC at P, and having radii 1·6 inches long. Measure the distances between the centres of all these circles.

5. ABCD is a square, of which AC is a diagonal. Bisect the angle BAC by the straight line AE, which meets BC in E. Prove that $AB + BE = AC$.

6. Prove that parallel tangents to a circle must touch the circle at the extremities of a diameter of the circle.

Paper 51.

1. Draw a circle of radius 0·9 inch; draw a line through the centre; on it take two points, each 2·3 inches from the centre, and draw the tangents from these points. What is the figure formed by the four tangents? Verify by suitable measurements.

2. ABC is an isosceles triangle having AB = AC. Produce BC both ways, and bisect the exterior angles so formed by straight lines which meet, when produced, in D. Join AD, and prove that AD bisects the angle BAC.

3. With centres O and O′ draw two circles to touch each other internally at A; from A draw a straight line cutting the circumferences in B and B′; and join OB and O′B′. Prove that OB is parallel to O′B′.

4. Describe a circle of radius 1·6″ to pass through two points which are 2·3″ apart.

5. Draw a triangle ABC in which BAC is the largest angle and ACB the smallest angle. Bisect each of the angles of the triangle, and let the bisectors meet in I. Prove that IA < IB < IC.

6. Show how to describe a circle that shall pass through two given points P and Q, and touch a given straight line AB, which is parallel to the line joining P and Q.

Paper 52.

1. Mark with a lead pencil round about one-third of the edge of a penny, so as to get an arc of a circle; find the centre of this circle and so complete the circle. Measure its diameter in centimetres and in inches.

2. BDEC is the base of an isosceles triangle ABC, and BD = EC. Join AD and AE and prove that ADE is an isosceles triangle.

3. Two circles whose centres are P and Q touch externally at A, and a straight line BAC is drawn through A to meet the circles in B and C. Prove that the radii PB and QC are parallel.

4. Draw a triangle ABC, having AB = 5·2 cms., AC = 8·1 cms. and $\angle A = 41°$. Describe a circle round the triangle and measure its radius.

5. ABCD is a parallelogram, and AP and CQ are drawn from the opposite angles A and C perpendicular to the diagonal BD. Prove that AP = CQ.

6. From a given point P outside a given circle show how to draw a straight line PAB to cut the circle, such that the intercepted chord AB may be equal to a given finite straight line MN, which is less than a diameter of the circle.

Paper 53.

1. Draw a circle of radius 2·5″ and in it place a chord 2″ from the centre. Calculate and measure the length of this chord.

2. Show by diagrams that any obtuse-angled triangle may be represented as the sum of two right-angled triangles, and also as the difference of two right-angled triangles.

3. P is any point, except the centre, within the circle ABC. Prove that the shortest chord that can be drawn through P will be perpendicular to the diameter through P.

4. Draw the circum-circle of the triangle whose vertices are the points (0, 3), (2, 0) and (– 1, 0) and find its radius.

5. Prove that any straight line drawn through the point of intersection of the diagonals of a parallelogram and terminated by a pair of opposite sides is bisected at that point.

6. Draw a quadrilateral ABCD in which AB = 3·2″, BC = 3·4″, CD = 1·6″, DA = 2·9″ and AC = 3·8″. Find the area of ABCD correctly to within one-tenth of a square inch.

Paper 54.

1. Draw an isosceles triangle ABC, whose vertical angle BAC = 51°. Describe a circle touching the equal sides AB, AC and having its centre in the base BC.

2. If any straight line cuts two concentric circles in the points A, B and C, D respectively, prove that AC = BD.

3. Prove that if the diagonals of a quadrilateral inscribed in a circle bisect each other, the quadrilateral must be rectangular.

4. Describe the circle which passes through the three points (2, 0), (0, 3), (−1, 0) and find the co-ordinates of its centre.

5. A straight rod of given length slides between two fixed straight lines at right angles to each other. Find the locus of the centre of the rod.

6. ABC is a scalene triangle and on its three sides equilateral triangles are described pointing outwards. Show, by drawing a careful figure, that, if circles are described round these equilateral triangles, the three circles will all pass through the same point O within the scalene triangle. Join OA, OB, OC and measure each of the angles BOA, AOC and COB.

SECTION X.

The Exercises and Riders in Section X. are based on the same Problems and Theorems as those in Section IX., but are somewhat more difficult.

Paper 55.

1. Describe a circle to pass through the points of intersection of the straight lines which join the following pairs of points: (1, 1) and (−2, −2), (0, 1) and (1, −1), (2, 4), and (2, 3). Measure its radius.

2. BAC is a triangle having BAC a right angle. AP is drawn perpendicular to BC, and AM bisects BC. Prove that the angle MAP is equal to the difference of the angles at B and C.

3. Show that there is always one line about which two given circles are symmetrical. Under what circumstances is there more than one such line?

4. O is the centre of a circle whose radius is 1·45″, and P is a point 2·75″ from O. Calculate correctly to $\frac{1}{100}$th of an inch the length of the tangent drawn from P to the circle, and verify your calculation by drawing a careful figure.

5. If M, N be the middle points of the opposite sides AB, CD of a parallelogram ABCD, prove that DM and BN trisect the diagonal AC.

6. If from any point A on the circumference of a circle two unequal chords AB and AC are drawn, prove that the smaller chord makes the greater angle with the diameter drawn through A.

Paper 56.

1. Inscribe a circle in the triangle whose angular points are (4, 0), (0, 5) and (1, 1), and find the co-ordinates of the centre of this circle.

2. ABC is a triangle having the side AB greater than the side BC, and M is the middle point of AC. Join BM, and prove that BY, the bisector of the angle ABC, will fall within the angle MBC.

3. Show that all circles which have their centres on a fixed straight line, and also pass through one fixed point not on this line, must pass through another fixed point.

4. Draw a circle of radius 1·4 cms., and in it inscribe a triangle ABC having AB = 1·6 cms. and AC = 2 cms. From A draw AP perpendicular to BC, and join A to O, the centre of the circle. Measure the angle PAO.

5. ABCD is a parallelogram and its diagonal CA is produced to E so that CA = AE. If the parallelogram BAEF is completed, prove that FA and AD are in one and the same straight line.

6. The escribed circle of the triangle ABC touches the side BC at P, and the sides AB and AC, when produced, at Q and R. Prove that AQ and AR are each equal to half the perimeter of the triangle ABC.

Paper 57.

1. Draw a triangle ABC having its sides 2·7″, 3·8″ and 4·4″ long. Describe the circumscribing circle of this triangle. Draw AP, BQ and CR perpendiculars to BC, CA and AB respectively. Through the points P, Q, R describe another circle. Measure the radius of each of these circles.

2. BAC is any acute angle, and P is any point within the angle. Show how to draw through P a straight line QPR which is terminated by AB and AC and has QP equal to PR.

3. If AB, CD are two unequal parallel chords of a circle, show by the method of symmetry, that the point of intersection of AC and BD, when produced, and the point of intersection of AD and BC, lie on the line of the same diameter.

4. Describe 3 circles, with radii 2 cms., 4 cms. and 6 cms. respectively, to touch each other externally. Join the centres of the three circles and measure the angles of the triangle so formed.

5. ABCDE is a regular pentagon. Join AC and AD, and prove that each of the angles ACD and ADC is double of the angle CAD.

6. AB is a fixed chord of a circle, and PQ is any diameter of the circle; prove that the sum of the perpendiculars drawn from P and Q to AB is constant for all positions of the diameter PQ.

Paper 58.

1. Draw a triangle ABC having its sides 2″, 3″ and 3·6″ long. Describe the circumscribing circle of this triangle. Bisect each of the sides of the triangle ABC at the points D, E, F. Through the points D, E, F describe another circle. Measure the radius of each of these circles.

2. A kite at A, flown by a boy at B, is vertically over a point C on the ground, which forms a horizontal plane in which B also lies. If AB = 505 feet and BC = 456 feet, calculate the height of the kite above the ground.

3. If two circles meet at a point on the straight line joining their centres, show that they do not meet at any other point.

4. In a triangle whose angles are 85°, 55° and 40° respectively, inscribe a circle whose centre is I. Join I to the points of contact of the sides of the triangle and measure the three angles so formed at the point I.

5. Trisect a given triangle by straight lines drawn through any given point in one of its sides.

6. Two tangents AP and BQ are drawn from the extremities of a diameter AB of a given circle, and a third tangent to the same circle cuts these two tangents in P and Q. Prove that PQ subtends a right angle at the centre of the circle.

Paper 59.

1. On a straight line 4·6 cms. long describe an arc of a circle containing an angle of 30°, without drawing a tangent.

2. ABC is an isosceles triangle, having the side AB equal to the side AC, and D is any point within the triangle such that the angle DAB is greater than the angle DAC. Prove that the angle DBA is greater than the angle DCA.

3. Find the radii of the circumscribed and inscribed circles of a triangle whose sides are 3, 4 and 5 cms. long respectively.

4. Draw a straight line, and a circle of radius 2·3″ having its centre 3·7″ from the line. Show how to construct a second circle of radius 1·8″ touching both the straight line and the first circle. How many such circles can be drawn?

5. Prove that the point of intersection of the diagonals of a rhombus is equidistant from its four sides.

6. ABCDEF is an irregular hexagon circumscribed about a circle. Prove that $AB + CD + EF = BC + DE + FA$ and verify by measuring the sides of your figure.

Paper 60.

1. If an area of 12 square inches represents an acre, what is the representative fraction?

2. ABC is any triangle, and straight lines BY, CZ are drawn perpendicular to any straight line passing through A. Prove that the middle point of BC is equidistant from Y and Z.

3. A square and an equilateral triangle are inscribed in the same circle. Compare their areas.

4. ABCD is a quadrilateral having AB = 2·6 cms., BC = 3·9 cms. and CD = 5·2 cms. Find the length of AD, if a circle can be inscribed in ABCD, and find the radius of this inscribed circle. Verify your calculation by drawing a complete figure and measuring the radius.

5. Points X, Y, Z are taken on BC, CA, AB, the sides of an equilateral triangle ABC, such that XYZ is also an equilateral triangle. Prove that AZ = BX = CY.

6. Draw any triangle ABC, and verify, by drawing, that the circle drawn through the middle points of the sides of the triangle ABC touches the inscribed circle and each of the three escribed circles of the triangle.

SECTION XI.

The Exercises and Riders in Sections XI. and XII., Papers 61 to 72, are based on the same Problems and Theorems as the previous Sections, with the addition of the following:

PROBLEMS.

15. To construct the common tangents to two given circles.

16. To construct regular figures of 3, 4, 5, 6 or 8 sides in or about a given circle.

THEOREMS.

35. In equal circles, or in the same circle, (i) if two arcs subtend equal angles at the centre, they are equal; (ii) conversely, if two arcs are equal, they subtend equal angles at the centre.

36. In equal circles, or in the same circle, (i) if two chords are equal, they cut off equal arcs; (ii) conversely, if two arcs are equal, the chords of the arcs are equal.

37. The angle which an arc of a circle subtends at the centre is double that which it subtends at any point on the remaining part of the circumference.

38. Angles in the same segment of a circle are equal; and if the line joining two points subtends equal angles at two other points on the same side of it, the four points lie on a circle.

39. The angle in a semi-circle is a right angle; the angle in a segment greater than a semi-circle is less than a right angle; and the angle in a segment less than a semi-circle is greater than a right angle.

40. The opposite angles of any quadrilateral inscribed in a circle are supplementary; and the converse.

Paper 61.

1. Draw two intersecting circles with radii 2 cms. and 3 cms. respectively, and place the two circles so that their common chord of intersection is the greatest possible.

2. AB and CD are two parallel chords of a circle. Prove that the arc AC is equal to the arc BD.

3. ABCD is a quadrilateral inscribed in a circle, and the side AB is produced to E. Prove that the exterior angle CBE is equal to the interior opposite angle CDA.

4. Two parallel chords of a circle are ·9″ and 1·2″ in length, and are 1·8″ apart. Find the radius of the circle, and draw a figure to verify your answer.

5. BAC is a given arc of a circle, and D is the middle point of the arc opposite to BAC. Prove that the bisectors of all angles in the segment BAC will pass through the point D.

6. How could you tell whether an angle in a segment of a circle was obtuse, right or acute without measuring the angle?

Paper 62.

1. Draw two circles with radii ·6″ and ·8″, and whose centres are 1·8″ apart. Draw the four common tangents to these circles, and measure the lengths of the tangents between the points of contact.

2. Find the locus of the centres of all circles which touch two given intersecting straight lines. What does this locus become if the two lines (*a*) intersect at infinity, or (*b*) are coincident?

3. ABCD is a cyclic quadrilateral, and two of its opposite sides, AB and DC, are produced to meet in L. Prove that the triangles LBC and LDA are equiangular.

4. Draw a circle of 1·5 cms. radius, and about it describe an equilateral triangle. Measure the length of a side of this triangle.

5. Draw two circles with the same centre O. Prove that all tangents drawn from any point on the outer circle to the inner circle are equal in length.

6. Under what conditions is it apparently possible to draw only 2, or only 3 common tangents to a circle? Draw figures to illustrate your answer.

Paper 63.

1. Describe a square of 3″ side, inscribe a circle in the square, and form an octagon by cutting off the corners with symmetrically placed tangents to the circle. Measure the length of each side of the octagon.

2. Given the base BC, and the sum of the base angles ABC and ACB, of a triangle ABC. Find the locus of its vertex A.

3. If the opposite sides AB and CD of a cyclic quadrilateral ABCD are parallel, prove that the diagonals DB and AC are equal.

4. Draw a circle with radius 1·6 cms. and construct the two tangents TP and TQ to the circle intersecting at an angle PTQ = 60°. Measure and calculate the distance of the point T from the centre of the circle.

5. Two given circles touch externally at P, and also touch a given straight line, one circle at Q and the other at R. Prove that the angle QPR = 90°.

6. Two circles intersect in P and Q, and two parallel straight lines are drawn through P and Q, terminated by the circumferences. Prove that these lines are equal in length.

Paper 64.

1. The sides of a triangle are 60 ft., 70 ft. and 80 ft. respectively. Find, by diagram drawn to scale, the lengths of the radii of the circumscribed and inscribed circles.

2. Prove that the largest rectangle that can be inscribed in a given circle is a square.

3. AB, CD are two chords of a circle which intersect at P within the circle. Prove that if O is the centre of the circle, the angle APD is equal to half the sum of the angles AOD and BOC.

4. Draw a circle of radius 1·8″ and divide its circumference into 9 equal parts. Complete the nonagon and calculate and measure the size of each of its angles.

5. Prove that the two circles, described on the equal sides of an isosceles triangle as diameters, intersect on the base of the triangle.

6. If a triangle be inscribed in a circle, prove that the sum of the angles in the three segments exterior to the triangle is equal to four right angles.

Paper 65.

1. Draw a circle of radius 1·5 inches, and in the circle inscribe a triangle having its base 2·4 inches long and one of its base angles 51°.

2. TP and TQ are two tangents to a circle whose centre is O. Prove that TO will bisect the chord PQ at right angles.

3. If two chords of a circle intersect at P without the circle, prove that the angle at P is equal to an angle at the centre of the circle standing on half the difference of the arcs intercepted by the two chords.

4. Draw a triangle whose sides are 26 mms., 32 mms. and 36 mms. long. Produce the sides of the triangle and draw the three escribed circles. Join their centres and measure the lengths of the sides of the triangle so formed.

5. AB and CD are two parallel chords of a circle, and AC and BD are joined and produced to meet in T. Prove that TA = TB. What does this prove if the chords AB and CD are coincident?

6. Two circles intersect at P and Q; prove that the angle between the two tangents at P is supplementary to the angle between the two radii drawn from P.

Paper 66.

1. Draw two circles with radii 2·4 cms. and 3·4 cms. respectively, and touching each other internally at A. On the circumference of the smaller circle find a point P such that AP = 4 cms., and draw the tangent at P, meeting the larger circle in Q and R. Measure the angles QAP and RAP.

2. If a quadrilateral be circumscribed to a circle, show that the sum of one pair of opposite sides is equal to the sum of the other pair of sides.

3. Find a point within a triangle at which the sides subtend equal angles. Is this always possible?

4. Draw a circle of radius ·7″ and circumscribe to it a regular pentagon. Measure the length of a side of the pentagon.

5. ABC is a triangle inscribed in a circle. Prove that the internal and external bisectors of the angle BAC will meet the circumference in two points at the extremities of a diameter.

6. Two circles intersect at A and B, and AP and AQ are diameters of the circles. Prove that the line PQ will pass through B.

SECTION XII.

The Exercises and Riders in Section XII. are based on the same Problems and Theorems as those in Section XI., but are somewhat more difficult.

Paper 67.

1. Draw a triangle ABC, whose sides are 4 cms., 6 cms. and 8 cms. long. Describe three circles, whose centres are A, B, C, to touch each other externally in pairs.

2. A chord of a given circle varies its direction while continuing to pass through a given point. Prove that the locus of the middle point of the chord is a circle. What are the greatest and the least possible values of the radius of this second circle?

3. Two circles intersect in A and B, and any two straight lines, PAR and QBS, are drawn through A and B, terminated by the circumferences. Prove that PQ and RS are parallel.

4. Draw two circles with radii 1·4 and 3·5 cms. respectively, the distance between their centres being 6·3 cms. Draw the four common tangents of these circles, and measure the lengths of the tangents between the points of contact.

5. The straight line AB is divided in C, so that $AC^2 = 2CB^2$; prove that AB + CB will be equal to the diagonal of the square described on AB.

6. ABC is a triangle inscribed in a circle, and the arcs BC, CA, AB are bisected at the points P, Q, R. Prove that the bisectors of the angles of the triangle ABC are perpendicular to the lines QR, RP, PQ.

Paper 68.

1. Draw a regular pentagon, having each side 1·3″ long, and in the pentagon inscribe a circle. Measure the radius of the circle in inches and in centimetres.

2. ABC is a triangle inscribed in a circle, and P is the middle point of the arc BC. From P, PM and PN are drawn perpendicular to AB and AC respectively. Prove that the triangles PAM, PAN are congruent, and that $AM = \frac{1}{2}(AB + AC)$.

3. D and E are any points on BC, a side of a triangle ABC. Circles are described round ABD and ACE and intersect in F. Join DF and EF, and produce DF and EF to meet AC in G and AB in H. Prove that AHFG is a cyclic quadrilateral.

4. Draw two equal circles of radii 1·6 cms., their centres being 4·2 cms. apart. Draw the four common tangents to these circles and measure the lengths of the tangents in centimetres and in inches.

5. Prove that if the sum of one pair of opposite sides of a convex quadrilateral is equal to the sum of the other two sides, a point can be found, which is equidistant from the four sides.

6. ABCD is a quadrilateral inscribed in a circle, and P, Q, R, S are the middle points of the arcs AB, BC, CD, DA. Prove that the lines PR and QS will intersect at right angles.

Paper 69.

1. Draw a straight line AB 2·5 cms. long, and on AB describe a quadrilateral whose angles taken in order are in the ratio 1 : 2 : 3 : 4, and having CD, the longest side of the quadrilateral, equal to 5 cms.

2. Prove that the straight lines which bisect any angle of a quadrilateral inscribed in a circle, and its opposite exterior angle, meet on the circumference of the circle.

3. Two equal circles touch externally in A. Prove that the sum of the squares on the two tangents drawn, one to each circle, from any point T, is equal to twice the square on TA.

4. Draw an isosceles triangle, having AB = AC = 6·8 cms., and BC = 4 cms. In BC make BD = 1·5 cms., and join AD. About the triangles ABD and ACD describe circles, and measure the distances of their centres from AB and AC respectively.

5. ABCD is a quadrilateral. Two of its opposite sides are bisected at X and Y, and its diagonals AC and BD are bisected at Z and W. Prove that XZWY is a parallelogram whose area is equal to half the difference of the areas of the triangles ABC and ABD.

6. Find the locus of a point, the tangents from which to a given circle always intersect at a constant angle.

Paper 70.

1. Show by a diagram that a floor can be paved with tiles that are regular octagons and squares.

2. ABC is an isosceles triangle, and on the base BC a segment of a circle is described which cuts the equal sides, AB and AC, in P and Q respectively. Prove that the triangles APC and AQB are equal in area.

3. ABC is a triangle inscribed in a circle, and the bisector of the angle BAC meets BC in D and the circum-circle in E. Prove that the triangles ABD and AEC are equiangular.

4. Draw two circles of radii 1·5 and 2 cms. respectively, intersecting at right angles at the points A and B. Through A draw any chord PAQ. Join PB and QB, and measure the angle PBQ.

5. Two circles intersect at A and B, and two chords PAQ and PBR are drawn. Prove that so long as P moves along the circumference of the same circle, the chord QR in the other circle will be of constant length.

6. ABC is a triangle inscribed in a circle. AD and BE, the perpendiculars from A and B to the opposite sides, intersect in P, and AD is produced to meet the circumference in L. Prove that PD = DL.

Paper 71.

1. Draw a circle of radius ·75″ and in it place two perpendicular chords of lengths 1·1″ and 1·2″ respectively. Construct the tangents at the extremities of each of these chords and let these tangents intersect in the points P, Q, R, S. Find the sum of the angles at the opposite points P and R.

2. BAC is a right-angled triangle, and BDEC is the square on the hypotenuse BC, and ACHK is the square on AC. Join AE and BH, intersecting in L. Prove that CL will bisect the angle ELH.

3. Show how to draw through a point O, external to a given circle, a straight line OAB, cutting the circle in A and B, so that OA = AB. When is this impossible?

4. Draw a straight line AB = 7 cms. long. Divide AB in extreme and mean ratio, so that AB : AP = AP : PB. On AB describe an isosceles triangle ACB, having AC = CB = AP. Measure the angle ACB.

5. TP, TQ are tangents to a given circle, whose centre is C. If CT intersects the chord PQ in N, prove that $CN \cdot CT = CP^2$.

6. Two circles cut one another at A and B. Show how to draw through A a straight line PAQ, terminated by the circumferences, so that PA = AQ.

Paper 72.

1. Construct a quadrilateral ABCD, having AB = 4 cms., BC = 2·5 cms., and the angles at B, A and C respectively 45°, 60° and 120°. Circumscribe the quadrilateral by a circle and measure its radius.

2. AC and BD are two diameters of a circle at right angles to each other, and EF is a chord parallel to AC. In AC take any point P and prove that $PE^2 + PF^2 = 2PB^2$.

3. ABCD is any quadrilateral, and the bisectors of each of its angles form another quadrilateral. Prove that this latter is a cyclic quadrilateral.

4. Draw two straight lines intersecting at an angle 49°. Describe four circles, each of radius 1·2″, touching both the intersecting lines.

5. ABCD is a cyclic quadrilateral, and AD and BC produced meet in Q, and AB and DC produced meet in P. If the circles described round QDC and PBC intersect in R, prove that P, R, Q are collinear points.

6. O is the centre, and AOB a diameter of a circle. On AO and OB as diameters, circles are described. Prove that the circle described so as to touch the large circle internally and the two smaller circles externally has a radius $\frac{1}{3}$rd the radius of the larger circle.

SECTION XIII.

The Exercises and Riders in Section XIII., Papers 73 to 78, are based on the same Problems and Theorems as the previous Sections, with the addition of the following :

THEOREMS.

41. If a straight line touch a circle, and from the point of contact a chord be drawn, the angles which this chord makes with the tangent are equal to the angles in the alternate segments.

42. If two chords of a circle intersect either inside or outside the circle the rectangle contained by the parts of the one is equal to the rectangle contained by the parts of the other.

43. If from a point without a circle a tangent and a secant be drawn to the circle, the square on the tangent is equal to the rectangle contained by the secant and the part of the secant outside the circle ; and conversely.

Paper 73.

1. Draw two straight lines AOC, BOD, such that AO = 1·8″, OC = 1·4″, BO = 1·2″, OD = 2·1″. Join AB, BC, CD, DA. Show (1) by multiplication, and (2) by measuring the angles BAD, and BCD, that ABCD is a cyclic quadrilateral.

2. Show how to describe a square whose area is 10 square inches, and hence find by measurement the square root of 10, correct to two decimal places.

3. Two circles touch externally at A, and through A two straight lines PAQ and RAS are drawn, meeting one circle in P and R, and the other circle in Q and S. Prove that the triangles PAS and RAQ are equal in area.

4. Draw a circle of radius 4·2 cms., and from a point O outside the circle draw a secant OPQ such that OP = 5·6 cms., and OQ = 12·6 cms. Draw a tangent OT to the circle and find by measurement and by calculation the length of OT.

5. AB is a given finite straight line, and CD is another straight line of unlimited length. Find a point P in CD such that the angle APB is a maximum.

6. If two circles touch externally, show that the square on their common tangent is equal to the rectangle contained by the two diameters of the circles.

Paper 74.

1. Draw a triangle ABC, whose sides are 2″, 3″ and 4″ long. Draw perpendiculars from each of the angular points to the opposite sides and let these perpendiculars meet in P. Describe circles round ABC and PAB, and measure the diameters of each of these circles.

2. Prove that if a polygon inscribed in a circle is equilateral it is also equiangular.

3. Show how to describe a circle to pass through two given points and to touch a given straight line.

4. Describe a circle of radius 2·1 cms., and draw an isosceles triangle of altitude 2·4 cms., such that the area of the isosceles triangle is equal to the area of the circle.

5. BAC is an angle in a circle standing on the arc BC, and the arc BC is bisected at P. If PM is drawn perpendicular to AB, prove that $MB = \frac{1}{2}(AB \sim AC)$.

6. ABCD is a parallelogram whose diagonals intersect in O. Prove that the circles which are described about the triangles AOB and COD touch each other.

Paper 75.

1. Draw a straight line AB 1·5″ long and on AB describe a triangle ABC, such that AC + BC = 2·5″, and the ∠ ACB = 60°. Find the area of this triangle.

2. If C is the middle point of an arc ACB, prove that AC bisects the angle between AB and the tangent at A.

3. Show how to describe a circle to touch two given straight lines and to pass through a given point.

4. Construct an isosceles triangle ABC, having BC = 1″, and each of the angles at B and C double of the angle at A. Describe a circle round ABC and draw a tangent to this circle at B to meet AC produced in D. Measure and calculate the length of BD and of CD, and the number of degrees in the angle at D.

5. ABCD is a quadrilateral inscribed in a circle, and the opposite sides of ABCD are produced to meet in P and Q. Prove that the bisectors of the angles P and Q are at right angles to each other.

6. A and C are two fixed points, and two straight lines AB and CD rotate round A and C with equal speed. If BA and DC produced meet in P, prove that the locus of P is a circle.

Paper 76.

1. Find geometrically the value of $\frac{17 \times 28}{19}$.

2. ABCD is a parallelogram of which the angle ABC is 120°. Prove that each of the circles described round ABC, ADC, passes through the centre of the other circle.

3. If two circles cut in A and B, and T is a point such that tangents drawn from T to the circles are equal, prove that TA, produced if necessary, must pass through B.

4. Draw three straight lines OA, OB, OC, meeting at O, such that the $\angle AOB = 38°$, and the $\angle BOC = 47°$. Show how to draw a straight line perpendicular to OC, and such that the part of it intercepted between OA and OB is 4 cms. long.

5. XY is a given straight line of unlimited length, and A and B are any two points on opposite sides of XY. Find the points in XY at which A and B subtend a right angle.

6. Prove that, if three circles intersect, their common chords are concurrent.

Paper 77.

1. Make an angle of 54°. On one arm of the angle take two points P and Q, distant respectively 1″ and 3″ from the apex, and through P and Q draw the circle which touches the other arm of the angle. Measure the radius of the circle so drawn.

2. Prove that the locus of a point P, which moves so that the sum of the squares on its distances from 4 given points, A, B, C, D is constant, is a circle whose centre is the middle point of the line joining the middle points of the diagonals of the quadrilateral ABCD.

3. Show how to draw through a given point a straight line which shall divide a given circle into two segments, such that the angle in one segment is 5 times the angle in the other segment.

4. Construct a sector of a circle of radius 1·5″, and having an angle of 75°. In this sector inscribe a circle and measure its radius.

5. In the triangle ABC the bisectors of the interior angles at B and C meet in I, and the bisectors of the exterior angles at B and C meet in I′. Prove that a circle can be described round BICI′, and that, if this circle touches AB, then AB = AC.

6. A is a fixed point, and P is any point on a given circle BCD. Prove that, if AP is produced to Q, so that AP = PQ, then Q lies on another fixed circle.

Paper 78.

1. Draw two circles of 3 cms. and 4 cms. radius respectively, having their centres 6 cms. apart. Find a point from which the tangents drawn to the two circles will be 1·5 cms. and 2·5 cms. long respectively.

2. ABC is a triangle inscribed in a circle and a line PQ is drawn parallel to the tangent at A cutting AB and AC at D and E. Prove that B, D, E, C are concyclic points.

3. Given the base and the vertical angle of a triangle; find the locus of its orthocentre.

4. Draw two concentric circles with radii 1·5″ and 2″ respectively. Construct a trapezium equal in area to the annular space between the two circles, the parallel sides of the trapezium being equal in length to the circumferences of the two circles.

5. Prove that the locus of the points of intersection of the pairs of tangents, drawn to a given circle at the extremities of all the chords passing through a given fixed point within the circle, is a straight line perpendicular to the diameter passing through the given point.

6. What is meant by the apothem of a regular polygon? Show how the apothem can be used to describe a triangle equal in area to a given regular polygon.

SECTION XIV.

The Exercises and Riders in Section XIV., Papers 79 to 84, are based on the same Problems and Theorems as the previous Sections, with the addition of the following:

PROBLEMS.

17. To construct a fourth proportional to three given straight lines.

18. To construct a mean proportional to two given straight lines.

THEOREMS.

44. The ratio of the areas of two triangles or parallelograms of the same altitude is equal to the ratio of their bases.

45. If a straight line is drawn parallel to one side of a triangle, the other two sides are divided proportionally; and the converse.

46. The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle; and likewise the external bisector externally.

47. If two triangles are equiangular, their corresponding sides are proportional; and the converse.

48. If two triangles have one angle of the one equal to one angle of the other, and the sides about these equal angles proportional, the triangles are similar.

49. The ratio of the areas of similar triangles is equal to the ratio of the squares on corresponding sides.

50. The rectangle contained by the diagonals of a convex quadrilateral inscribed in a circle is equal to the sum of the rectangles contained by pairs of opposite sides.

Paper 79.

1. Draw a circle of 1″ radius. Inscribe a square in the circle and circumscribe a square about the circle. Find the area of each of these squares and of the circle.

2. ABCD is a trapezium having the side AD parallel to the side BC. Prove that any straight line drawn parallel to AD will divide the sides AB and CD proportionally.

3. AB is a straight line which is divided internally and externally at P and Q in the same ratio. Prove that

$$\frac{1}{AP}+\frac{1}{AQ}=\frac{2}{AB}.$$

4. Take two straight lines, AB = 2·4 cms. and CD = 4·6 cms. Find geometrically the Arithmetic, Geometric and Harmonic Means of AB and CD, and verify your figure by calculating the means.

5. XOY is any given acute angle, and P is any point on either OX or OY. From P draw PN perpendicular to the other arm of XOY, and prove that the ratio $\frac{PN}{OP}$, that is, the sine of the angle XOY, is constant for all positions of P.

6. ABC is a right-angled triangle, having the angle BAC a right angle, and AD is drawn perpendicular to BC. Prove that the triangles BAC, BAD, and DAC are similar, and that $AD^2 = BD \cdot DC$, $BA^2 = BD \cdot BC$ and $CA^2 = CD \cdot CB$.

Paper 80.

1. Draw a circle of 2 cms. radius. Inscribe an equilateral triangle in the circle and describe an equilateral triangle about the circle. Find the area of each of these triangles and of the circle.

2. ABC is a triangle, and the side BC is bisected at P. The angles APB, APC are bisected by the straight lines PQ, PR, meeting AB, AC in Q and R. Prove that QR is parallel to BC.

3. ABCD is a cyclic quadrilateral and the opposite sides AB and DC, when produced, meet in L. Prove that LAD and LCB are similar triangles.

4. Draw a triangle ABC, having AB = 3 cms., BC = 4 cms. and CA = 5 cms. Construct a similar triangle DEF, of which the shortest side DE = 2·3 inches.

5. ABC is a triangle inscribed in a circle, of which AD is a diameter. If AP is drawn perpendicular to BC, prove that AB . AC = AP . AD.

6. Prove that the points (0, 0), (3, 2), (5, $3\dot{3}$) lie on one straight line, and find in what ratio the line is divided.

Paper 81.

1. Describe a regular pentagon on a line 1″ long. Inscribe a circle in and describe a circle about this pentagon. Find the areas of both the circles and of the pentagon.

2. Two circles touch each other internally at O, and a straight line PRQ touches the inner circle at R and meets the outer circle in P and Q. Prove that OP : OQ = PR : RQ.

3. Prove that in similar triangles the medians have the same ratio to one another as the sides of the triangles.

4. On AB and DE, two given straight lines 2·4 cms. and 3·2 cms. long, describe two triangles ABC and DEF, having $\angle A = \angle D = 60°$, and $\angle B = \angle E = 45°$. Find the areas of the triangles and the ratio of the areas.

5. ABC is a triangle and BM, one of its medians, is bisected at N. Prove that BC will be trisected by AN produced, and by the line drawn through M parallel to AN.

6. ABC is any triangle and AD, BE, CF, the perpendiculars to the opposite sides of the triangle, meet in P. Prove that the triangles ADF and PDE are similar.

Paper 82.

1. Construct a rectangle having an area of 5 square inches and its sides in the ratio 2 : 3.

2. Prove that two diagonals of a regular pentagon, which intersect within the pentagon, divide each other in extreme and mean ratio.

3. The point A bisects the arc of a circle and the chord BC is the base of the arc. A straight line APQ is drawn, cutting BC in P, and meeting the circumference again in Q. Prove that $AP \cdot AQ = AB^2 = AC^2$.

4. Draw a straight line $AB = 4{\cdot}58''$ long and divide it at P and Q, so that

$$AP : PQ : QB = 1{\cdot}7 : 2{\cdot}3 : 3{\cdot}8.$$

5. ABC is a triangle inscribed in a circle, of which AD is a diameter. Prove that twice the area of the triangle ABC : the rectangle $AC \cdot BC = AB : AD$.

6. The sides of a triangle are $2{\cdot}3''$, $3{\cdot}4''$ and $4{\cdot}5''$, and the sides of another triangle are 2·3 cms., 3·4 cms. and 4·5 cms. Find the ratio of the areas of the two triangles and find the area of each triangle.

Paper 83.

1. Construct an equilateral triangle, having an area of 4 square inches.

2. AB and CD are two finite straight lines which are not parallel and do not intersect. Show how to draw, without producing AB and CD, a straight line that would pass through the intersection of AB and CD if produced.

3. The bisector of the angle A of a triangle ABC meets the base BC in D and the circum-circle in E. Prove that AB . AC = AD . AE.

4. Draw a straight line 4·5″ long to represent 15 miles, and on it construct a diagonal scale to show miles and furlongs. Complete and figure the scale.

5. In the triangles ABC, DEF the $\angle A$ = the $\angle D$; prove that

$$\frac{\triangle ABC}{\triangle DEF} = \frac{AB \,.\, AC}{DE \,.\, DF}.$$

6. In a given triangle ABC inscribe a square, so that two of its vertices are on BC and the other two vertices are on AB and AC.

Paper 84.

1. Describe a regular hexagon on a line 1″ long; and then describe a square equal in area to the hexagon.

2. Prove that the difference between two unequal sides of a triangle is either less than or equal to the difference between the perpendiculars let fall on them from the opposite angles. Distinguish between the two cases.

3. Find the locus of a point which moves so that the ratio of its distances from two given intersecting straight lines is constant.

4. On a straight line 10 cms. long describe a semi-circle. In the semi-circle construct a square, having two of its corners on the diameter and two on the circumference. Explain your construction.

5. OC is a fixed line and A a given point on OC. The point R moves on the circumference of a fixed circle whose centre is C, and the straight line OR is divided at P, so that $\frac{OP}{OR} = \frac{OA}{OC}$. Prove that P describes a circle whose centre is A.

6. If a straight line LMN cuts the sides BC, CA, AB of the triangle ABC in L, M, N respectively, then the ratio

$$\frac{CL \cdot AM \cdot BN}{MC \cdot NA \cdot LB} = 1.$$

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